

## 7. SUBJECT DETAILS

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### **7.1.1 OBJECTIVES AND RELEVANCE**

***The main objective of Linear ODE with variable coefficients and series solutions is to evaluate many differential equations for getting series solutions.***

***Several applications include Heat, Wave and Laplace's equation with cylindrical symmetry can be solved in terms of Bessel's functions, with spherical symmetry by Legendre polynomials.***

***Functions of complex variable however are useful in the study of fluid mechanics, thermodynamics and electric field and also useful in diverse branches of Science and Engineering.***

### **7.1.2 SCOPE**

***This subject gives the idea about Linear ODE with variable coefficients and series solutions is to evaluate many differential equations for getting series solutions. The general theory of complex variables enables engineers in solving problems like the evaluation of improper integrals by using Cauchy's Integral formulae and the Residue theorem; to deal with the rates of change applying the theory of analytical functions. The methodology of mapping is used to map complicated regions conformally onto simpler, standard regions such as circular disks, half planes and strips for which boundary value problems are easier.***

### **7.1.3 PREREQUISITES**

***Requires knowledge of ordinary differential equations, elementary calculus (differentiation and basic integration) and two dimensional geometry.***

### **7.1.4.1 JNTU SYLLABUS**

#### **UNIT-I**

#### **OBJECTIVE**

***Transforming the given variable coefficient equation (Cauchy's and Lagrange's) into the one with constant coefficients. Identifying ordinary points, singular points and regular singular points for the given ODE. Finding the series solution around a regular singular point. Solve the given ODE with variable coefficients by Frobenius method and test the convergence of its series solution.***

**SYLLABUS**

**Linear ODE with variable coefficients and series solutions (second order only): Equations reducible to constant coefficients-Cauchy's and Lagrange's differential equations. Motivation for series solutions, Ordinary point and Regular singular point of a differential equation , Transformation of non-zero singular point to zero singular point. Series solutions to differential equations around zero, Frobenius Method about zero .**

**UNIT-II****OBJECTIVE**

**Algebraic and transcendental functions together constitute the elementary functions, special functions such as Legendre polynomials, Chebycher's polynomials.**

**SYLLABUS**

**Special Functions : Legendre's Differential equation, General solution of Legendre's equation, Legendre**

**polynomials Properties: Rodrigue's formula – Recurrence relations, Generating function of Legendre's polynomials – Orthogonality. Bessel's Differential equation, Bessel functions properties: – Recurrence relations, Orthogonality, Generating function , Trigonometric expansions involving Bessel functions.**

**UNIT-III****OBJECTIVE**

**Complex analysis is concerned with complex analytic functions. In this chapter we explain a check for analyticity based on so-called Cauchy-Riemann Equations. Cauchy-Riemann Equations help us determine whether the complex value function is differential (or) not.**

**In this chapter we study the most important elementary complex functions such as Trigonometric, exponential, Logarithmic functions which generalize familiar real functions known from calculus.**

**SYLLABUS**

**Complex Functions –Differentiation and Integration : Complex functions and its representation on Argand**

**plane, Concepts of limit Continuity, Differentiability, Analyticity, Cauchy-Riemann conditions, Harmonic functions – Milne – Thompson method. Line integral – Evaluation along a path and by indefinite integration – Cauchy's integral theorem – Cauchy's integral formula – Generalized integral formula.**

**UNIT-IV****OBJECTIVE**

**This chapter is devoted mainly to series representations of analytic functions. The theorems presented in this chapter guarantee the existence of such representations .**

**Integration in complex plane is important for two reasons and they are (i) In applications there occur real integrals that can be evaluated by complex integration where usual methods of real integral calculus fail (ii) Some basic properties of analytic functions can be established by complex integration but would be difficult to prove by other methods.**

**SYLLABUS**

**Power series expansions of complex functions and contour Integration:** Radius of convergence - Expansion in Taylor's series, Maclaurin's series and Laurent series. Singular point -Isolated singular point - pole of order m - essential singularity. Residue - Evaluation of residue by formula and by Laurent series - Residue theorem. Evaluation of integrals of the type

$$(a) \text{ Improper real } \int_{-\infty}^{\infty} f(x)dx \qquad (b) \int_{\alpha}^{\alpha+2\pi} f(\sin\theta, \cos\theta)d\theta$$

**UNIT-V****OBJECTIVE**

To learn transform a given function from z - plane to w - plane.  
To identify the transformations like translation, magnification, rotation and reflection and inversion.  
To learn the Properties of bilinear transformations.

**SYLLABUS**

**Conformal mapping:** Transformation of z-plane to w-plane by a function, Conformal transformation. Standard transformations- Translation; Magnification and rotation; inversion and reflection, Transformations like  $\Theta z$  ,  $\log z$  ,  $z^2$ , and Bilinear transformation. Properties of Bilinear transformation, determination of bilinear transformation

**7.1.4.2 GATE SYLLABUS****UNIT-I****UNIT-II**

**Special Functions.**

**UNIT-III****Cauchy's integral theorem and integral formula.****Analytic functions****7.1.4.3 IES SYLLABUS****Not applicable****7.1.5 SUGGESTED BOOKS****TEXT BOOKS**

1. Advanced Engineering Mathematics by Kreyszig, John Wiley & Sons.
2. Higher Engineering Mathematics by Dr. B.S. Grewal, Khanna Publishers.

**REFERENCE BOOKS**

- 1) Complex Variables Principles And Problem Sessions By A.K.Kapoor, World Scientific Publishers
- 2) Engineering Mathematics-3 By T.K.V.Iyengar and B.Krishna Gandhi Etc
- 3) A Text Book Of Engineering Mathematics By N P Bali, Manesh Goyal
- 4) Mathematics for Engineers and Scientists, Alan Jeffrey, 6<sup>th</sup> Edit. 2013, Chapman & Hall/CRC
- 5) Advanced Engineering Mathematics, Michael Greenberg, Second Edition. Person Education

**7.1.6 WEBSITES**

1. [www.wikipedia.org](http://www.wikipedia.org)
2. [www.grad.gatech.edu](http://www.grad.gatech.edu) (Georgia institute of technology)
3. [www.eng.ufl.edu](http://www.eng.ufl.edu) (University of Florida)
4. [www.iitk.ac.in](http://www.iitk.ac.in)
5. [www.iitd.ernet.in](http://www.iitd.ernet.in)
6. [www.psgtech.edu](http://www.psgtech.edu)
7. [www.iisc.ernet.in](http://www.iisc.ernet.in)
8. [www.math.helsinki.fi](http://www.math.helsinki.fi)
9. [www.mathworld.wolfram.com](http://www.mathworld.wolfram.com)
10. [www.efunda.com/math/legendre/index.cfm](http://www.efunda.com/math/legendre/index.cfm)

**7.1.7 EXPERTS' DETAILS****INTERNATIONAL**

1. **Dr. A. G. Sergeev,**  
**Research Area : Complex Analysis,**  
**Steklov Mathematical Institute,**  
**e-mail: sergeev@mi.ras.ru.**
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**NATIONAL**

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**REGIONAL**

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2. **Prof. D. Ram Murthy,**  
**Research Area : Complex Analysis,**  
**Department of Mathematics,**  
**Osmania University, Hyderabad-500007.**

**7.1.8 JOURNALS****INTERNATIONAL**

1. **Complex variables and elliptic equations- Taylor & Francis group (NA)**
2. **Integral transforms and Special functions - Taylor & Francis group (NA)**
3. **Complex Analysis and Operator theory (NA)**

4. ***International e-journal of Engineering Mathematics-Theory and application (NA)***
5. ***Applied Numerical Mathematics. (NA)***
6. ***Mathematical models and methods in Applied sciences. (NA)***
7. ***Journal of Mathematical Analysis and Applications. (NA)***
8. ***Complex variables theory and Applications. (NA)***
9. ***Mathematics of Computation. (NA)***

**NATIONAL**

1. ***Journal of interdisciplinary mathematics. (NA)***
2. ***Proceedings of the Indian Academy of Sciences:Mathematical sciences. (A)***
3. ***Journal of Information and optimization sciences. (NA)***
4. ***Journal of Discrete mathematical sciences and Cryptography. (NA)***
5. ***Indian Journal of pure and applied mathematics. (NA)***
6. ***Indian Journal of Mathematics (A)***

**( NA-Not available in library, A-Available in library)**

**7.1.9 FINDINGS AND DEVELOPMENTS**

1. ***“Integral means of Analytic Functions”, Daniel .G and Jose A.P., Annales Academiae, Scientarium Fennicae Mathematica, 29, pp. 459-69, 2004.***
2. ***“Two results of uniqueness of conformal mappings”, Steven G. K., Complex variables theory and applications 50, No. 6, pp. 427 - 32, May 2005.***
3. ***“Complex dynamics, value distributions and potential theory”, Yusuke O., Annales Academiae Scientiarum Fennicae Mathematica, 30, pp. 303 - 11, 2005***
4. ***Unique range set for certain Meromorphic functions, Ming-Liang Fang and Indrajit Lahiri, Indian Journal of Mathematics, Vol. 45, Issue No. 2, Pages 141 - 150, 2004.***

**7.1.10 SESSION PLAN**

Sl. No.	Topics in JNTU syllabus	Modules and Sub modules	Lecture	Suggested books	Remarks
<b>UNIT-I</b>					
1	<b>Linear order differentiation with variable coefficients</b>	Equations reducible to constant coefficients	L 1	T1-Ch4, R2-Ch1	
		Cauchy's and Lagrange's differential equations.	L2,3	T1-Ch4, R2-Ch1	
		Motivation for series solutions	L4,5	T1-Ch4, R2-Ch1	
		Ordinary point and Regular singular point of a differential equation	L6,7	T1-Ch4, R2-Ch1	
		Transformation of non-zero singular point to zero singular point	L8	T1-Ch4, R2-Ch1	
		Series solutions to differential equations around zero, Frobenius Method about zero.	L9,10	T1-Ch4, R2-Ch1	
<b>UNIT-II</b>					
2	<b>Special Functions</b>	Legendre's Differential equation	L11	T1-Ch4, R2-Ch2	
		General solution of Legendre's equation	L12	T1-Ch4, R2-Ch2	
		Legendre polynomials Properties: Rodrigue's formula - Recurrence relations	L13	T1-Ch4, R2-Ch2	
		Generating function of Legendre's polynomials – Orthogonality.	L14,15	T1-Ch4, R2-Ch2	
		Bessel's Differential equation, Bessel functions properties: – Recurrence relations	L16,17	T1-Ch4, R2-Ch2	
		Orthogonality, Generating function	L18	T1-Ch4, R2-Ch2	
		Trigonometric expansions involving Bessel functions.	L19,L20	T1-Ch4, R2-Ch2	
<b>UNIT-III</b>					
3	<b>Complex functions – differentiation and integration</b>	Complex functions and its representation on Argand plane	L21	T1-Ch12, R2-Ch3	
		Concepts of limit Continuity, Differentiability, Analyticity	L22	T1-Ch12, R2-Ch3	
		Cauchy-Riemann conditions	L23,24	T1-Ch12, R2-Ch3	
		Harmonic functions – Milne – Thompson method	L25,26	T1-Ch12, R2-Ch3	
		Line integral – Evaluation along a path and by indefinite integration	L27,28	T1-Ch13, R2-Ch4	
		Cauchy's integral theorem – Cauchy's integral formula – Generalized integral formula.	L29,30	T1-Ch13, R2-Ch4	
<b>UNIT-IV</b>					
4	<b>Power series expansions of complex functions and contour integration</b>	Radius of convergence	L31	T1-Ch14, R2-Ch5	
		Expansion in Taylor's series	L32,33	T1-Ch14, R2-Ch5	
		Maclaurin's series and Laurent series	L34,35	T1-Ch14, R2-Ch5	
		Singular point, Isolated singular point, Isolated singular point, Pole of order m	L36,37	T1-Ch15, R2-Ch5	
		Residue – Evaluation of residue by formula and by Laurent series – Residue theorem.	L38,39	T1-Ch15, R2-Ch6	

Sl. No.	Topics in JNTU syllabus	Modules and Sub modules	Lecture	Suggested books	Remarks
		Evaluation of integrals of the type $\int_{-\infty}^{\infty} f(x)dx$	L40,41	T1-Ch15, R2-Ch6	
		$\int_{\alpha}^{\alpha+2\pi} f(\sin \theta, \cos \theta)d\theta$	L42,43	T1-Ch15, R2-Ch6	
<b>UNIT-V</b>					
7	<b>Conformal mapping</b>	Transformation by $e^z$ , $\ln z$ , $z^2$ , $z^n$ (n positive integer)	L44	T2-Ch18, R2-Ch7	
		$\sin z$ , $\cos z$ , $z + a/z$	L45,46	T2-Ch18, R2-Ch7	
		Translation	L46,47	T2-Ch18, R2-Ch7	
		rotation	L48	T2-Ch18, R2-Ch7	
		inversion and bilinear transformation	L49,50	T2-Ch18, R2-Ch7	
		fixed point	L51,52	T2-Ch18, R2-Ch7	
		cross ratio properties	L53	T2-Ch18, R2-Ch7	
		invariance of circles and cross ratio	L54,55	T2-Ch18, R2-Ch7	
		determination of bilinear transformation mapping 3 given points	L56,57	T2-Ch18, R2-Ch7	

**7.6.11 STUDENT SEMINAR TOPICS**

- 1. Special functions and their applications.**
- 2. Analytic Functions and their applications.**
- 3. Residence Theorem and their applications.**

**7.1.12 ASSIGNMENT QUESTIONS**

**UNIT-I**

- 1) Solve in series the equation  $y^{11} - xy = 0$  about  $x=0$
- 2) Solve in series the equation  $y^{11} - xy^1 + y = 0$  about  $x=0$
- 3) Solve in series the equation  $4xy^{11} + 2y^1 + y = 0$
- 4) Solve in series the equation  $xy^{11} + y + xy = 0$
- 5) Solve in series the equation  $x(1-x)y^{11} - xy^1 - y = 0$

**UNIT-II**

- 1) Express  $J_3(x)$  in terms of  $J_0(x)$  and  $J_1(x)$ .
- 2) Prove that  $J_{-n}(x) = (-1)^n J_n(x)$  where  $n$  is positive integer.
- 3) Show that  $x^3 = \frac{2}{5} p_3(x) + \frac{3}{5} p_1(x)$ .
- 4) Express  $x^3 + 2x^2 - x - 3$  in terms of Legendre polynomials.
- 5) Prove that  $\cos x = J_0 - 2J_2 + 2J_4 - \dots$

**UNIT-III**

- 1) Find the analytic function whose real part is  $\frac{x}{x^2 + y^2}$
- 2) State and prove the Cauchy-Riemann equations.
- 3) Show that  $f(z) = |z|^2$  is not analytic.
- 4) Evaluate  $\int_{(0,0)}^{(1,3)} 3x^2 y dx + (3^3 - y^2) dy$  along the curve  $y=x$ .
- 5) State and prove the Cauchy integral formula.

**UNIT-IV**

1) Find the Taylor's series expansion for  $f(Z) = \frac{Z}{Z+2}$  about  $Z=1$

2) Find the Laurent series expansion of the function  $f(Z) = \frac{1}{Z^2 - 3Z + 2}$  in the region  $1 < |Z| < 2$

3) Explain about the different types of singularities.

4) State and prove the Cauchy residue theorem.

5) Evaluate  $\int_0^{\infty} \frac{dx}{(x^2 + a^2)^2}$

**Unit-v**

1) Define the conformal mapping.

2) Under the transformation  $W = \frac{1}{Z}$  find the image of the circle  $|Z - 2i| = 2$

3) Find the image of the line  $x=4$  in  $Z$ -plane under the transformation  $W = Z^2$

4) Find the bilinear transformation which maps the points  $Z=1, i, -1$  onto the points  $W=1, 0, -i$

5) Find the bilinear transformation which maps the points  $(i, 0)$  into the points  $(0, i)$

### Question Bank

#### Unit - I

#### Unit- II

1. State and prove generating function for  $J_n(x)$ .
  - a) Prove that  $(2n+1)P_n(x) = P_{n+1}'(x) - P_{n-1}'(x)$ .
  - b) Prove that  $\int_{-1}^1 x P_n(x) P_{n-1}(x) dx = \frac{2n}{4n^2 - 1}$ . [15] (Dec 13)
3. Prove that  $J_4(x) = \left(\frac{48}{x^3} - \frac{8}{x}\right)J_1(x) + \left(1 - \frac{24}{x^2}\right)J_0(x)$  (May-13)
4. Prove that  $x^2 J_n''(x) = (n^2 - n - x^2)J_n(x) - x J_{n+1}(x)$ .
5. Show that  $\frac{n}{x} J_n(x) + J_n'(x) = J_{n-1}(x)$  (Dec 12)
6. Express  $J_2(x)$  in terms of  $J_0(x)$  &  $J_1(x)$ . (Dec 11)
7. S.T.  $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}} \left\{ \frac{3-x^2}{x^2} \sin x - \frac{3}{x} \cos x \right\}$  (Dec 11)
8. i. Express the following in terms of Legendre polynomials  $4x^3 - 2x^2 - 3x + 8$ .  
 ii. Evaluate  $\int_{-1}^1 x^2 U_3(x) dx$ . (Dec 11)
9. i. P.T.  $J_n(x) = 0$  has no repeated roots except at  $x = 0$  (Nov 10)  
 ii. P.T.  $\frac{d}{dx} \{x J_1(x)\} = x J_0(x)$
10. i. S.T.  $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}} \{1/x \sin x - \cos x\}$  (Nov 10)  
 ii. S.T.  $\int_0^1 x^3 (1 - \sqrt{x})^5 dx = 2\beta(8,6)$
11. P.T.  $x J_n'(x) = n J_n(x) - (x) - x J_{n+1}(x)$  (Nov 10)
12. Show that  $J_4(x) = \left(\frac{48}{x^3} - \frac{8}{x}\right) J_1(x) + \left(1 - \frac{24}{x^2}\right) J_0(x)$  (Feb 08)
13. ii. Prove that  $\frac{d}{dx} [x^{-n} J_n(x)] = -x^{-n} J_{n+1}(x)$

Creation is a drug I can't do without.

- Cecil B. DeMille

- iii. When n is an integer, show that  $J_{-n}(x) = (-1)^n J_n(x)$  (Feb 08)
  
- 14. Show that when 'n' is a positive integer,  $J_n(x)$  is the coefficient of  $z^n$  in the expansion of  $\exp\left\{\frac{x}{2}\left(z - \frac{1}{z}\right)\right\}$ . (Feb 08, Nov 07, May 01)
  
- 15. Prove that  $J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2 \cdot 4^2} - \frac{x^6}{2^2 \cdot 4^2 \cdot 6^2} + \dots$  (Nov 07)
  
- 16. Prove that  $(2n+1)(1-x^2)P_n^1(x) = n(n+1)(P_{n+1}(x) - P_{n-1}(x))$  (Feb, Nov 07)
  
- 17. Prove that  $\frac{d}{dx}(xJ_n J_{n+1}) = x(J_n^2 - J_{n+1}^2)$  (Feb 07)
  
- 18. Prove that  $J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x} - \cos x\right)$  (Nov 10)
  
- 19. Prove that  $\frac{n}{x} J_n(x) + J_n^1(x) = J_{n-1}(x)$  (Feb 07)
  
- 20. Prove that  $\Gamma\left(\frac{1}{n}\right) \Gamma\left(\frac{2}{n}\right) \Gamma\left(\frac{3}{n}\right) \dots \Gamma\left(\frac{n-1}{n}\right) = \frac{(2\pi)^{\frac{n-1}{2}}}{n^{\frac{1}{2}}}$  (May 11, Feb 07, May 06)
  
- 21. Prove that  $J_0^2 + 2(J_1^2 + J_2^2 + \dots) = 1$  (Nov 06, 03)
  
- 22. Prove that  $J_2 - J_0 = 2J_0^1$  (Nov, May 06, Apr 05)
  
- 23. Show that  $4J_n^1(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)$  (May 11, May 06)
  
- 24. When n is a positive integer show that  $J_n(x) = \frac{1}{n} \int_0^\pi \cos(n\theta - x \sin \theta) d\theta$  (May 06)
  
- 25. Write  $J_{\frac{5}{2}}(x)$  in finite form. (May 06)
  
- 26 a) Prove that  $\int_{-1}^1 x^2 P_{n+1} P_{n-1} dx = \frac{2n(n+1)}{(2n+1)(2n-1)(2n+3)}$
- b) **Prove that**  $(1-x^2)T_n^1(x) = nT_{n-1}(x) - nxT_n(x)$  (May-13)
  
- 27. a) Express  $x^3 - 2x^2 + x + 2$  in terms of Legendre polynomial.

The merit of originality is not novelty; it is sincerity.

- Thomas Carlyle

- b) Show that  $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$ . **(Dec12)**
28. a) Prove that  $\frac{1+z}{z\sqrt{1-2xz+z^2}} - \frac{1}{z} = \sum_{n=0}^{\infty} [P_n(x) + P_{n+1}(x)]z^n$
- b) Prove that  $(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$  **(Dec12)**
29. i. P.T.  $P_n^1(-1) = (-1)^{n-1} \frac{n(n+1)}{2}$
- ii. Write  $T_2(x) + T_1(x) + T_0(x)$  as a polynomial. **(Dec11)**
5. i. P.T.  $(1-x^2)p_n^1(x) = (n+1)\{xp_n(x) - P_{n+1}(x)\}$
- ii. Express the following in terms of Legendre polynomials  $1+x-x^2$ . **(Dec11)**
30. i. S.T.  $x^3 = \frac{2}{5} P_3(x) + \frac{3}{5} P_1(x)$ .
- ii. Express  $f(x) = 2x+10x^3$  in terms of Legendre polynomials. **(Dec11)**
31. i. Express  $x^3+3x^2+4x+3$  in terms of Legendre polynomials.
- ii. Evaluate  $\int_0^1 x(1-x^2)^{-\frac{1}{2}} U_4(x) dx$ . **(May 11)**
32. If  $f(x) = \begin{matrix} 0 & ; & 1 < x < 0 \\ x & ; & 0 < x < 1 \end{matrix}$  then **(Nov 10)**  
 S.T.  $f(x) = \frac{1}{4} P_0(x) + \frac{1}{2} P_1(x) + \frac{5}{16} P_2(x) - \frac{3}{32} P_4(x) + \dots$
33. i. P.T.  $\int_{-1}^1 (P_n^1)^2 dx = n(n+1)$
- ii.  $P_n - P_{n-1} = (2n-1)P_{n-1}$  **(Nov 10)**
34. i. Evaluate  $\int_{-1}^1 x^4 (1-x^2)^{-1/2} T_2(x) dx$  **(Nov 10)**
- ii. Prove that  $\int_{-1}^1 x^2 P_{n-1}(x) P_{n+1}(x) dx = 0$
35. i. Prove that, for all  $x^7 = \frac{116}{429} P_7(x) + \frac{8}{9} P_5(x) + \frac{14}{33} P_3(x) + \frac{1}{3} P_1(x)$
- ii. Show that  $\int_{-1}^1 x^k P_n(x) dx = 0$  for  $k = 0, 1, 2, n-1$  **(Nov 10)**
36. Show that  $\int_{-1}^1 (1-x^2) (P_n^1)^2 dx = \frac{2n(n+1)}{(2n+1)}$  **(Feb 08)**





37. Establish the formula  $P'_{n+1}(x) - P'_{n-1}(x) = (2n + 1) P_n(x)$  (Feb 08)
38. i. Prove that  $\int_{-1}^1 P_m(x)P_n(x)dx = \begin{cases} 0; m \neq n \\ \frac{2}{2n+1}; m = n \end{cases}$  (Feb 08, Nov 07, Jan 03)
- ii.  $\int_{-1}^1 x^n P_n(x)dx = \frac{2^{n+1} \cdot (n!)^2}{(2n+1)!}$  (Nov 07)
39. Prove that  $\int_{-1}^1 x P_n(x)P_{n-1}(x) = \frac{2n}{(4n^2-1)}$  (Nov 07)
40. Prove that  $(1-x^2)P'_n(x) = (n+1)[xP'_n(x) - P'_{n+1}(x)]$  (Feb, Nov 07)
41. Express  $x^3 + 2x^2 - x - 3$  in terms of Legendre Polynomials (Feb 07)
42. Prove that  $\int_{-1}^1 (x^2 - 1)P_{n+1}P'_n dx = \frac{2n(n+1)}{(2n+1)(2n+3)}$  (Feb 07)
43. Prove that  $\frac{1}{\sqrt{1-2tx+t^2}} = P_0(x) + P_1(x)t + P_2(x)t^2 + \dots$  (May 11, Nov, May 06, Jan 03)
44. Prove that  $P_n(0) = 0$  for n odd and  $P_n(0) = \frac{(-1)^{\frac{n}{2}} n!}{2^n (\frac{n}{2}!)^2}$  if n is even. (Nov, May 06, Apr 05)
45. Show that  $x^4 = \frac{8}{35} P_4(x) + \frac{4}{7} P_2(x) + \frac{1}{5} P_0(x)$  (Nov 06)
46. Show that  $x^3 = \frac{2}{5} P_3(x) + \frac{3}{5} P_1(x)$ . (May 06)
47. Using Rodrigue's formula prove that  $\int_{-1}^1 x^m P_n(x)dx = 0$  if  $m < n$ . (Nov 10)

**UNIT-III**

1. Show that the function  $f(x) = \begin{cases} \frac{x^3 y(y-ix)}{x^6 + y^2}, z \neq 0 \\ 0, z = 0 \end{cases}$  is not differentiable at the origin. (Dec 13)
- a) Evaluate  $\int_0^1 (x^2 + iy) dz$  along the path  $y = x$  and  $y = x^2$ .
- 2 b) Use Cauchy's Integral formula to evaluate  $\int_C \frac{z-1}{(z+1)^2(z-2)} dz$ , where 'C' is the circle  $|z-i|=2$ . [15] (Dec 13)
- 3 a) Find the analytic function whose real part is  $e^{2x}(x \cos 2y - y \sin 2y)$

*Man was made at the end of the week's work when God was tired.*

- Mark Twain

- b) Show that the function defined by  $f(z) = \frac{x^3(1+i) - y^3(1-i)}{(x^2 + y^2)}$  at  $z \neq 0$ , and  $f(0) = 0$  is continuous and satisfies C.R equations at the origin, but  $f'(0)$  does not exist. (May 13)
4. a) Prove that  $U = x^2 y^2$ ;  $V = \frac{y}{x^2 + y^2}$  are harmonic functions of (x,Y) but are not harmonic conjugates.  
 b) Find an analytic function whose real part is  $e^x(x \sin y - \cos y)$  (Dec12)
5. a) find the analytical function whose real part is  $r^2 \cos 2\theta + r \sin \theta$ .  
 b) If  $f(z)$  is an analytic function of  $z$ , Prove that  $\left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] |f(z)|^2 = 4f'(z)^2$  (Dec12)
6. i. If  $W = \phi + i\psi$  represents the complex potential for an electric eld &  $\psi = x^2 - y^2 + \frac{x}{x^2 + y^2}$ , determine the function  $\phi$ .  
 ii. If  $f(z) = u+iv$  is an analytic function of  $z$  &  $u-y = e^x(\cos y - \sin y)$  find  $f(z)$  in terms of  $z$ . (Dec11)
7. i. Find whether the function  $u = \log |z|^2$  is harmonic. If so, find the analytic function whose real part is  $u$ .  
 ii. Separate the real and imaginary parts of  $i^{\log(1+i)}$ .
8. i. If  $f = u + iv$  is analytic in a domain  $D$  and  $uv$  is constant in  $D$ , then prove that  $f(z)$  is constant.  
 ii. Find the general and principal values of  $\log(1+i\sqrt{3})$ . (Dec11)
9. i. P.T. The function  $f(z)$  defined by  $f(z) = \begin{cases} \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}; & (z \neq 0) \\ 0; & (z = 0) \end{cases}$  is continuous and the C-R equations are satisfied at the origin, yet  $f'(0)$  does not exist. (Dec11)
10. ii. S.T. The function  $u = 1/2 \log(x^2 + y^2)$  is harmonic & find its conjugate. (May 11)
11. i. S.T. the real & imaginary parts of the function  $w = \log z$  satisfy the C-R equations when  $z$  is not zero.  
 ii. S.T.  $f(z) = z + 2\bar{z}$  is not analytic anywhere in the complex plane. (May 11)
12. S.T. both the real & imaginary parts of an analytic function are harmonic. (May 11)
13. i. S.T.  $\tan^{-1} z = \frac{1}{2} \log \frac{i+z}{i-z}$

Ideas are the root of creation.

- Ernest Dimnet

- ii. Find the roots of  $\sin z = \cosh 4$  (Nov 10)
- 14. i. Prove that  $\tan^{-1} z = \left[ \log \frac{(i+z)}{(i-z)} \right]$  (Nov 10)
- ii. ....  $r^{-4} \cos 4\theta$
- 15. Find the analytic function whose real part is  $\frac{y}{x^2 + y^2}$  (Nov 10)
- 16. i. S.T. the real & imaginary parts of the function  $w = \log z$  satisfy the C - R equations when  $z$  is not zero.
- ii. S.T.  $f(z) = z + 2 \bar{z}$  is not analytic anywhere in the complex plane. (Nov 10)
- 17. i. Find the analytic function whose real part  $\mu = e^{2x} [x \cos 2y - y \sin 2y]$
- ii. Find whether  $f(z) = \sin x \sin y - i \cos x \cos y$  is analytic or not
- 18. i. Find the modulus and argument of  $i^{\log(1+i)}$
- ii. Show that an analytic function of constant module is constant. (May 11)
- 19. i. Prove that the function  $f(z) = \sqrt{|xy|}$  is not analytic at the origin even though the C - R equations are satisfied thereat.
- ii. Find the analytic function whose real part is  $y / (x^2 + y^2)$ . (Nov 09, 08)
- 20. i. Derive Cauchy Riemann equations in polar coordinates. (Nov 09, Feb 08, May 05, Jan 03)
- ii. Prove that the function  $f(z) = \text{conjugate of } z$  is not analytic at any point.
- 21. i. Show that  $w = z^n$  ( $n$ , a positive integer) is analytic and find its derivative. (Nov 09, May 05)
- ii. Prove that  $\mu = e^{x^2 - y^2}$  is a harmonic function and find its harmonic conjugate.
- 22. Find whether  $f(z) = \frac{x - yi}{x^2 + y^2}$  is analytic or not. (Nov 09, May 02)
- 23. Find the general and principal values of (i)  $\log_{\epsilon}(1 + \sqrt{3}i)$  (ii)  $\log_{\epsilon}(-1)$  (Nov 09, Feb 08, Nov 04)
- 24. Separate the real and imaginary parts of  $\tan hz$ . (Nov 09, May 06)
- 25. Separate the real and imaginary parts of  $\sin hz$ . (Nov 09, May 06)
- 26. i. Find the real and imaginary parts of  $\text{Cot } z$ .
- ii. Prove that  $e^{\bar{z}}$  is analytic
- iii. Find  $e^z$  and  $|e^z|$  if  $z = 4\pi(2+i)$  (Nov 09)
- 27. Solve  $\tan h + 2 = 0$  (Nov 09)
- 28. If  $\tan(x+iy) = A+iB$  show that  $A^2 + B^2 - 2B \cot h^2 Y + 1 = 0$  (Nov 11)

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Great effort is required to arrest decay and restore vigor. One must exercise proper deliberation, plan carefully before making a move, and be alert in guarding against relapse following a renaissance. - I Ching

29. i. Show that the function defined by  $f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}$  at  $z \neq 0$  and  $f(0)=0$  is continuous and satisfies CR equations at the origin but  $f'(0)$  does not exist **(Nov 08, 06, Feb 07, May 01)**
- ii. ....  $x \cos y$ .
30. i. Show that  $f(z) = \frac{xy^2(x+iy)}{x^2 + y^2}$ ,  $(x, y) \neq (0,0)$  and 0 at  $(x,y)=(0,0)$  is not analytic at  $z = 0$  although CR equations are satisfied at the origin **(Nov 08, Feb 07)**
- ii. If  $w = \phi + i\psi$  represents the complex potential for an electric field and  $\psi = 3x^3y - y^3$  find  $\phi$ .
31. The necessary and sufficient conditions for the function  $f(z) = u(x, y) + i v(x, y)$  to be analytic in the region R, are **(Nov 08)**
- i.  $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x}, \frac{\partial v}{\partial y}$  are continuous functions of  $x$  and  $y$  in R.
- ii.  $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$
32. i. Prove that
- a.  $i^n = e^{-i(n-1)\pi/2}$
- b.  $\log i^n = -(2n + 1/2)\pi$
- ii. If  $\tan(x + iy) = A + iB$ , Show that  $A^2 + B^2 + 2A \cot 2x = 1$ . **(Nov 08)**
33. i. Separate into real and imaginary parts of  $\cosh(x + iy)$ . **(Nov 08)**
- ii. Find all the roots of the equation
- a.  $\sin z = \cosh 4$
- b.  $\sin z = i$ .
34. i. Find the real part of the principal value of  $i^{\log(1+i)}$  **(Nov 08)**
- ii. Separate into real and imaginary parts of  $\sec(x + iy)$ .
35. i. Separate into real and imaginary parts of  $\coth z$ .
- ii. If  $\tan \log(x + iy) = a + ib$  where  $a^2 + b^2 \neq 1$  prove that  $\tan \log(x^2 + y^2) = \frac{2a}{1 - a^2 - b^2}$  **(Nov 08, May 05)**
36. If  $\sin(\alpha + i\beta) = x + iy$  then prove that  $\frac{x^2}{\cosh^2 \beta} + \frac{y^2}{\sinh^2 \beta} = 1$  and  $\frac{x^2}{\sin^2 \alpha} - \frac{y^2}{\cos^2 \alpha} = 1$ . **(Feb 08)**
37. Determine the analytic function  $f(z) = u + iv$  given that  $3u + 2v = y^2 - x^2 + 16x$ . **(Feb 08)**
38. Test for analyticity at the origin for  $f(z) = \frac{x^3y(y - ix)}{x^6 + y^2}$  for  $z \neq 0$ . **(Feb 08, May 06)**

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*I am opposed to millionaires, but it would be dangerous to offer me the position.*

- Mark Twain

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= 0  
for  $z=0$ .

39. Find all values of  $z$  which satisfy (i)  $e^z = 1 + i$  (ii)  $\sin z = 2$ . **(Feb 08, May 06, Nov 03)**
40. If  $w = f(z)$  is an analytic function, then prove that the family of curves defined by  $u(x,y) = \text{constant}$  cuts orthogonally the family of curves  $v(x,y) = \text{constant}$ . **(Feb 08, May 05)**
41. Show that  $f(x,y) = x^3y - xy^3 + xy + x + y$  can be the imaginary part of an analytic function of  $z = x + iy$ . **(Feb 08, May 05)**
42. Prove that  $\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |\text{Real } f(z)|^2 = 2 |f'(z)|^2$  where  $w = f(z)$  is analytic. **(Feb 08, May 05)**
43. Find  $f(z) = u + iv$  given that  $u + v = \frac{\sin 2x}{\cosh 2y - \cos 2x}$  **(Feb 08, Jan 03)**
44. Derive Cauchy Riemann equations in cartesian co-ordinates. **(Feb 07, Nov 06, May, Jan 03)**
45. If  $(x + iy)^{\frac{1}{3}} = a + ib$  then prove that  $4(a^2 - b^2) = \frac{x}{a} + \frac{y}{b}$  **(Feb 07)**
- 46 a) Evaluate  $\int_0^{1+i} (x^2 - iy) dz$  along i) The straight line  $y = x$  ii) Along  $y = x^2$
- b) Evaluate  $\int_c \frac{(z+4)dz}{(z^2 + 2z + 5)}$  where  $C$  is  $|z + 1 - i| = 2$  **(May 13)**
- 47.a) Evaluate  $\int_{(1,1)}^{(2,8)} (x^2 + ixy) dz$  along  $x = t$  and  $y = x^3$ .
- b) Using Cauchy's integral formula, evaluate  $\int_c \frac{z dz}{(z-1)(z+3)}$  where  $C: |z| = 1.5$ . **(Dec12)**
48. a) Evaluate  $\oint_c \frac{z-1}{(z+1)^2(z-2)} dz$  where  $C: |z-i|=2$
- b) Evaluate  $\int_c \frac{z^4}{(z+1)(z-1)^2} dz$  Where 'C' is the ellipse  $9x^2 + 4y^2 = 36$ . **(Dec12)**
49. i. Evaluate  $\int_{-\infty}^{\infty} \frac{x^2}{(1-x^2)(x^2+4)} dx$

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"When students are frustrated or get involved in put-down behavior, Random Acts of Kindness works beautifully."

- Author: Kathy, Teacher, CA

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- ii. Evaluate by Residue theorem  $\int_c \frac{\sin z}{z^6} dz$  where  $C: |z|=2$  **(Dec11)**
50. i. Evaluate  $\int_{(0,0)}^{(1,1)} ((x-y^2)dx + 2xydy)$  along  $x_0$  curve  $y = x$
- ii. Evaluate  $\int_{z=0}^{z=1+i} \{x^2 + 2xy + i(y^2 - x)\} dz$  along  $y = x^2$  **(Dec 11)**
51. i. Evaluate  $\int_c \frac{z^3 + z^2 + 2z - 1}{(z-1)^3} dz$  where  $C: |z|=3$
- ii. Evaluate  $\int_c \frac{z^4}{(z+1)(z-i)^2} dz$  where 'C' is the ellipse  $9x^2+4y^2=36$  **(Dec 11)**
52. Evaluate  $\int_c \frac{z-3}{z^2 + 2z + 5}$  where C is
- i.  $|z|=1$   
 ii.  $|z+1-i|=2$   
 iii.  $|z+1+i|=2$
- ii. Evaluate  $\int_c \frac{5z^2 - 3z + 2}{(z-1)^3} dz$  where c is any simple closed curve enclosing  $z = 1$ . **(Dec11)**
53. i. Evaluate  $\int_c \frac{3z^2 + 7z + 1}{z + 1} dz$  where  $C: |z + i| = 1$
- ii. Evaluate  $\int_c \frac{z^2 - z + 17z + 1}{z - 1} dz$  where  $C: |z| = \frac{1}{2}$  taken in anticlockwise sense **(Dec 11)**
54. i. Evaluate  $\int_c (z^2 + 3z + 2)dz$  where C is the arc of the cycloid  $x=a(q + \sin)$ ,  $y= a(1-\cos q)$  between the points  $(0,0)$  &  $(ap, 2a)$   
 ii. Evaluate  $\int_c (z^2 + 3z)dz$  along the straight line from  $(2,0)$  to  $(2,2)$  and then from  $(2,2)$  to  $(0,2)$ . **(May 11)**
55. The only singularities of a single valued function  $f(z)$  are poles of order 1 and 2 at  $z = -1$  and  $z = 2$  with residues at these poles  $i$  and  $2$  respectively. If  $f(0) = 7/4$ ,  $f(1) = 5/4$ , determine the function  $f(z)$ . **(May 11)**
56. Let 'C' denotes the boundary of the square whose sides lie along the lines  $x = \pm 2$ ,  $y = \pm 2$  where 'C' is described in the positive sense evaluate the following integrals **(May 11)**

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*Be careful about reading health books. You may die of a misprint.*

- Mark Twain

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- i.  $\int_C \frac{\sin(z/2)}{(z-x_0)^2} dz$  ( $|x_0| < 2$ )
  - ii.  $\int_C \frac{\cosh z}{z^4} dz$
57. From the integral  $\int_0^\pi \frac{dz}{z+1}$  S.T.  $\int_0^\pi \frac{1+i \cos \theta}{17+8 \cos \theta} - 0$  where  $C: |z|=1$ .
- ii. If  $C$  is a closed curve described in +ve sense and  $f(z_0) = \int_C \frac{z^4 dz}{(z-z_0)^4}$  show that  $f(z_0) = 8\pi i z_0$  is where  $z_0$  is a point inside 'C' and  $f(z_0) = 0$  if  $z_0$  lies outside 'C'. **(May 11)**
58. i. Evaluate  $\int_{1-i}^{2+i} (2x + iy + 1) dz$  along the straight line joining (1,-i) & (2,i)
- ii. If  $C$  is the boundary of the square with vertices at the points  $z = 0, z = 1, z = 1+i$  and  $z = i$  show that  $\int_C (3z + 1) dz = 0$  **(Nov 10)**
59. i. Evaluate  $\int_C (x + y) dx + x^2 y dy$  along  $y=3x$  between (0,0) and (3,a)
- ii. Evaluate  $\int_C e^z dz$  where  $C: |z|=1$  **(Nov 10)**
60. i. Evaluate  $\int_0^{3+i} z^2 dz$  along the line  $x=2y$
- ii. Use Cauchy's integral formula to evaluate  $\oint \frac{e^z}{(z+2)(z+1)^2} dz$  where  $C$  is the circle  $|z|=3$ .
61. i. a. Prove that  $\int_C \frac{dz}{z-a} = 2\pi i$  where  $C$  is  $|z-a|=r$  **(Nov 09, 08, 06)**
- b. Evaluate  $\oint_C (z-a)^n dz$ ; for  $n = -1, n \neq -1$ .  $n$  is integer.
  - ii. State and prove Cauchy's integral theorem.
62. Evaluate  $\int_{(0,0)}^{(1,1)} (3x^2 + 4xy + ix^2) dz$  along  $y = x^2$ . **(Nov 09, 06, 04, Feb 08)**
63. i. Evaluate  $\int_0^{1+i} (x^2 + iy) dz$  along the path  $y = x$  and  $y = x^2$ . **(Nov 09, 08)**
- ii. Evaluate, using Cauchy's integral formula  $\int_C \frac{e^{2z}}{(z-1)(z-2)} dz$ , where  $c$  is the circle  $|z|=3$ .
64. Evaluate  $\oint_C \frac{dz}{(z-a)^n}$  if  $z = a$  is point inside a simple closed curve  $C$  and  $n$  is integer. **(Nov 09, 06, May 00)**

*Clothes make the man. Naked people have little or no influence on society.*

- Mark Twain

65. Evaluate using Cauchy's theorem  $\int_c \frac{z^3 e^{-z} dz}{(z-1)^3}$  where c is  $|z-1| = 1/2$  using Cauchy's integral formula. (Nov 09, May 06)
66. i. Evaluate  $\int_0^{3+i} z^2 dz$ , along  
 a. the line  $y = x/3$   
 b. the parabola  $x = 3y^2$
- ii. Evaluate  $\int_c \frac{z^3 - 2z + 1}{(z-i)^2}$  where C is  $|z| = 2$  by using Cauchy's Integral Formula. (Nov 08, Jan 03)
67. i. Evaluate  $\int_0^{1+i} (x^2 - iy)$  along  $y = x^2$  and  $y = x$  (Nov 10)
- ii. Use Cauchy's integral formula to evaluate  $\oint_c \frac{\sin^2 z}{(z - \frac{\pi}{6})^3} dz$  where c is the circle  $|z| = 1$ .
68. i. Evaluate  $\int_C \frac{(z^3 - \sin 3z) dz}{(z - \frac{\pi}{2})^3}$  with C:  $|z| = 2$  using Cauchy's integral formula.
- ii. Evaluate  $\int_C \frac{dz}{e^z (z-1)^3}$  where C:  $|z| = 2$  Using Cauchy's integral theorem. (Feb 08)
69. Prove Cauchy's theorem. (Feb 08, 07)
70. Evaluate  $\int_c \frac{\log z dz}{(z-1)^3}$  where c:  $|z-1| = (1/2)$ , using Cauchy's Integral Formula. (Nov 10)
71. Evaluate using Cauchy's integral formula  $\int_c \frac{(z+1)dz}{z^2 + 2z + 4}$  where c:  $|z+1+i| = 2$ . (Nov 07, 06, May 06)
72. Evaluate using Cauchy's integral formula  $\int_c \bar{z} dz$ , from  $z = 0$  to  $4 + 2i$  along the curve C given by  
 i.  $z = t^2 + it$   
 ii. Along the line  $z = 0$  to  $z = 2$ : and then from  $z = 2$  to  $z = 4 + 2i$ . (Nov 07, May 06)
73. i. Find  $f(z)$  and  $f(3)$  if  $f(a) = \int_c \frac{(2z^2 - z - 2)dz}{(z-a)}$  where C is the circle  $|z| = 2.5$  using Cauchy's integral formula.
- ii. Evaluate  $\int_C \log z dz$  where C is the circle  $|z| = 1$  using Cauchy's integral formula. (Nov 07)
74. i. Evaluate  $\int_{-2+i}^{2+i} z^3 dz$  using Cauchy's integral formula along  $y = x$ .
- ii.  $\int (x+y)dx + ix^2$  along  $y = x^2$  from  $(0,0)$  to  $(3,9)$ .

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*Ethical axioms are found and tested not very differently from the axioms of science. Truth is what stands the test of experience.*  
 - Albert Einstein

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- iii. Evaluate  $\int_{-1+i}^{2+i} (x^2 - y^2 + i xy) dz$  using Cauchy's integral formula along  $y=x^2$ . (Nov 07)
75. i. Evaluate using Cauchy's integral formula  $\int_c \frac{(z+1)dz}{z^2 + 2z + 4}$  where  $C: |z + 1 + i| = 2$ .
- ii. Evaluate  $\int_c \bar{z} dz$  from  $z = 0$  to  $4 + 2i$  along the curve  $C$  given by  
 a.  $z = t^2 + it$   
 b. Along the line  $z = 0$  to  $z = 2$ : and then from  $z = 2$  to  $z = 4 + 2i$ . (Nov 07)
76. If  $F(a) = \int_c \frac{3z^2 + 7z + 1}{z - a} dz$ , where  $C$  is  $|z| = 2$  find  $F(1)$ ,  $F(3)$ ,  $F^{11}(1 - i)$  using Cauchy's Integral formula (Feb 07)
77. Evaluate  $\int_c \frac{z^3 - \sin z}{(z - \frac{\pi}{2})^3} dz$  with  $C: |z| = 2$  using Cauchy's integral formula (Feb 07)
78. Evaluate  $\int_c \frac{dz}{e^z (z - 1)^3}$  where  $C: |z| = 2$  using Cauchy's integral formula (Feb 07)
79. Evaluate  $\int_c \frac{dz}{z^2 e^z}$  where  $C: |z| = 1$  (Feb 07)
80. Evaluate using Cauchy's integral formula  $\int_0^{1+i} z^2 dz$  along  $y = x^2$  (May 11, Feb 07)
81. Evaluate  $\int_c \frac{\log z}{(z - 1)^3} dz$  where  $C: |z - 1| = \frac{1}{2}$  (Feb 07)

**UNIT-IV**

- a) Find the Taylor's series expansion of  $e^z$  about  $z = 3$ .
1. b) Expand  $f(z) = \frac{z^2 - 4}{z^2 + 5z + 4}$  in the region  $1 < |z| < 4$ . (Dec 13)
- a) Evaluate  $\int_c \frac{4 - 5z}{z(z - 1)(z - 2)} dz$ , where 'C' is the circle  $|z| = \frac{3}{2}$  using Residue theorem.
2. b) Evaluate by contour Integration  $\int_0^\infty \frac{dx}{1 + x^2}$ . [15] (Dec 13)

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*In the beginning the Universe was created. This has made a lot of people very angry and been widely regarded as a bad move.*  
 - Douglas Adams

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- 3 a) Expand  $\frac{z}{(z+1)(z+2)}$  About  $Z = 2$ .
- b) Expand  $\frac{4z+3}{(z-3)(z+2)}$  in the annular region between  $|z|=2$  and  $|z|=3$  (May 13)
4. a) Find a Taylor Series expansion of a function  $f(z) = \frac{1}{(z-1)(z-3)}$  about the point  $z=4$ . Obtain the region of convergence. (Dec12)
- b) Determine the poles and residue at each pole of the function  $f(z) = \frac{z^2}{(z-1)^2(z-2)}$ .
4. a) Find the Taylor's expansion for the function  $f(z) = \frac{1}{(1+z)^2}$  with centre at  $-i$ .
- b) Find the Laurent's series of  $\frac{1}{z^2 - 4z + 3}$  for  $1 < z < 3$ . (Dec12)
5. State and prove Taylor's Theorem of complex function  $f(z)$ . (Dec11)
6. i. Represent the function  $f(z) = \frac{1}{z(z+2)^3(z+1)^2}$  in Laurent series with in  $\frac{5}{4} \leq |z| \leq \frac{7}{4}$
- ii. Dene for a complex function (z)
- i. Isolated Singularity
  - ii. Removable Singularity
  - iii. Essential singularity
- (Dec11)
7. Expand  $\frac{7z-2}{(z+1)z(z-2)}$  about the point  $z = -1$  in the region  $1 < |z+1| < 3$  as Laurent's series. (Dec11)
8. i. For the function  $f(z) = \frac{2z^3+1}{z^2+z}$  find
- (a) Find the Taylor's series expansion of about  $z=3$ .
- (b) Explain  $f(z) = \cos z$  in Taylor's series about  $z = \frac{\pi}{2}$  (Dec11)
9. Expand  $f(z) = \frac{z+3}{z(z^2-z-2)}$  in powers of  $z$ .
- i. With in the unit circle about the origin
  - ii. With in the annular region between the concentric circles about the origin having radii 1 and 2 respectively.
  - iii. The exterior to the circle of radius 2. (May 11)

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*In those days spirits were brave, the stakes were high, men were real men, women were real women and small furry creatures from Alpha Centauri were real small furry creatures from Alpha Centauri.* - Douglas Adams

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10. Expand the function  $f(z) = \frac{4z+4}{z(z-3)(z+2)}$  in powers of  $z$ , when **(May 11)**
- $|z| \leq 1$
  - $1 \leq |z| \leq 2$
  - $|z| > 2$
11. Expand  $f(z) = \frac{z+3}{z(z^2-z-2)}$  in powers of  $z$ .
- With the in the unit circle about the origin **(Nov 10)**
  - With in the annular region between the concentric circle about the origin having radii 1 and 2 respectively
  - The exterior to the circle of radius 2
12. For the function  $f(z) = \frac{2z^2}{z^2+z}$  find
- A tailors expansion valid in the neighborhood of the point 'i'.
  - A Laurent's series valid within the annulus of which centre is origin **(Nov 10)**
13. i. Represent the function  $f(z) = \frac{1}{z(z+2)^3(z+1)^2}$  in Laurent series iwth in  $\frac{5}{4} \leq |z| \leq \frac{7}{4}$
- Define for a complex function (z)
    - Isolated Singularity
    - Removable Singularity
    - Essential Singularity **(Nov 10)**
14. Obtain the Laurent's series expansion of  $f(z) = \frac{e^z}{z(1-3z)}$  About  $z = 1$  **(Nov 09)**
15. i. Expand  $f(z) = \frac{1+2z}{z^2+z^3}$  in a series of positive and negative powers of  $z$ . **(Nov 09, 08)**
- Expand  $e^z$  as Taylor's series about  $z = 1$ .
16. i. Find Laurent's series for  $f(z) = \frac{1}{z^2(1-z)}$  and find the region of convergence. **(Nov 09, 08, May 06)**
- Expand  $\frac{1}{z^2-3z+2}$  in the region (i)  $0 < |z-1| < 1$  and (ii)  $1 < |z| < 2$  **(Nov 09, 08, Feb 07)**
17. i. State and derive Laurent's series for an analytic function  $f(z)$ . **(Nov 09, Feb 08, Nov 07, May 06)**
- Obtain Taylor's in the expansion of  $\frac{e^z}{z(z+1)}$  about  $z=2$
18. Expand  $f(z) = \frac{1}{z^2-z-6}$  about (i)  $z = -1$  (ii)  $z = 1$  **(Nov 09, 06)**
19. Obtain Taylor series to represent the function  $\frac{z^2-1}{(z+2)(z+3)}$  in the region  $|z| < 2$ . **(May 11, 06, Nov 09)**

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*Never hold discussions with the monkey when the organ grinder is in the room.*

- Sir Winston Churchill

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20. i. Expand  $\log(1-z)$  when  $|z| < 1$  (Nov 08)  
 ii. Determine the poles of the function  
 a.  $\frac{z}{\cos z}$   
 b.  $\cot z$ .
21. i. Expand  $f(z) = z-1/z+1$  in Taylor's series about the point  $z = 0$  and  $z = 1$ . (Nov 08)  
 ii. Determine the poles of the function  $\frac{1-e^{2z}}{z^4}$
22. Expand the Laurent series of  $\frac{z^2-1}{(z+2)(z+3)}$  for  $1 < |z| < 3$ . (Feb 08)
23. Evaluate  $f(z) = \frac{z}{z^2+1}$  where  $C$  is  $|z| = 3/2$ . (Feb 08)
24. Expand  $\log z$  by Taylor series about  $z = 1$  (Feb 08, 07, Nov 07)
25. Expand  $\frac{1}{(z^2+1)(z^2+2)}$  in positive and negative powers of  $z$  if  $1 < |z| < \sqrt{2}$  (Feb 08, 07, Nov 07)
26. Obtain all the Laurent series of the function  $\frac{7z-2}{(z+1)z(z-2)}$  about  $z = -2$  (Feb 08, 07, Nov 07)
27. Find the Taylor series of  $\frac{z}{z+2}$  about  $z = 1$ , also find the region of convergence. (Feb 08, Apr 05)
28. Find the Laurent expansion of  $\frac{1}{z^2-4z+3}$ , for  $1 < |z| < 3$ . (Nov 07, May 06)
29. Expand  $f(z) = \frac{(z-2)(z+2)}{(z+1)(z+4)}$  in the region (a)  $1 < |z| < 4$  (b)  $|z| < 1$  (Feb, Nov 07)
31. Find the poles and the residues at each pole of  $f(z) = \frac{z}{z^2+1}$ . (Nov 07)
32. Find the Laurent series expansion of the function (Nov 07, 06, 03)  
 i.  $\frac{z^2-1}{z^2+5z+6}$  about  $z = 0$  in the region  $2 < |z| < 3$ .  
 ii.  $\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$  in the region  $3 < |z+2| < 5$ . (May 11)
33. State and prove Taylor's theorem. (Feb 07, Nov, May 06, Nov 04, May 02)

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*You would win a man to your cause, first convince him that you are his sincere friend.*

- Abraham Lincoln

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34. Show that when  $|z+1| < 1, z^{-2} = 1 + \sum_{n=1}^{\infty} (n+1)(z+1)^n$ . ((May 11, Nov 06, 04, 03)

35. a) Evaluate  $\int_0^{2\pi} \frac{d\theta}{5-3\cos\theta}$  by contour integration in complex plane. (May 13)

36. a) Evaluate  $\int_0^{\pi} \frac{1}{3+2\cos\theta} d\theta$  by Contour integration in the complex plane.

b) Evaluate  $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+4)} dx$  by method of complex variables. (Dec12)

37. a) Evaluate  $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$  using residue theorem.

b) Evaluate by Residue theorem  $\int_C \frac{z-1}{(z+1)^2(z-2)} dz$  where C:  $|z-1|=2$ . (Dec12)

38. Find the bilinear transformation which maps  $z_1 = 1, 2i; z_2 = 2 + i; z_3 = 2 + 3i$  in to the points  $w_1 = 2 + 2i; w_2 = 1 + 3i; w_3 = 4$  respectively. Find the fixed and critical points. (Dec11)

39. i. Evaluate  $\int_0^{\infty} \frac{1}{1+x^6} dx$

ii. Evaluate by Residue theorem  $\int_C \frac{4z^2 - 4z + 1}{(z-2)(z^2+4)} dz$  C:  $|z|=1$ . (Dec11)

40. i. By the method of contour integration prove that  $\int_0^{\infty} \frac{\cos mx}{a^2+x^2} dx = \frac{\pi}{2} e^{-ma}$

ii. Evaluate by Residue theorem  $\int_C \frac{z^2}{(z-1)^2(z-2)} dx$  where  $|z|=3$ . (Dec11)

41. i. Evaluate  $\int_0^{\infty} \frac{\sin x}{x^2+4x+5} dx$

As been my experience that folks who have no vices have very few virtues.

- Abraham Lincoln

- ii. Evaluate by Residue theorem  $\int_c \frac{z-1}{(z+1)^2(z-2)} dz$  where  $C:|z-i|=2$  **(Dec11)**
42. i. Evaluate  $\int_0^{\infty} \frac{\log x}{1+x^2} dx$
- ii. Find the Residues of  $f(z) = \frac{1}{z(e^z-1)}$ . **(May 11)**
43. i. Using complex variable techniques evaluate  $\int_0^{2\pi} \frac{\sin^2 \theta d\theta}{5-4 \cos \theta}$
- ii. The only singularities of a single valued function  $f(z)$  are poles of order 2 and 1 at  $z=1$  &  $z=2$  with residues of these poles as 1 and 3 respectively. If  $f(0) = 3/2$ ,  $f(-1) = 1$ , determine the function. **(May 11)**
- i. Find the residue of  $\frac{1}{(z - \sin z)}$  at  $z = 0$
- ii. Evaluate  $\int \frac{(z-3)dz}{(z^2 + 2z + 5)}$  Where  $C$  is  $|z+1-i|=2$  **(Nov 10)**
44. i. Find the image of the triangle with vertices at  $i$ ,  $1+i$ ,  $1-i$  in the  $z$ -plane, under the transformation  $e^{\frac{5\pi i}{3}} \cdot (z-2+4i)$
- ii. Find the image of the infinite strip,  $0 < y < \frac{1}{2}$  under the mapping function  $w = \frac{1}{z}$  **(Nov 10)**
45. i. Evaluate  $\int_c \frac{z^4 + z^2 + 2z - 1}{(z-1)^3} dz$  where  $C: |z|=3$
- ii. Evaluate  $\int_c \frac{z^4}{(z+1)(z-i)^2} dz$  where 'C' is the Ellipse  $9x^2 + 4y^2 = 36$  **(Nov 10)**
46. i. Using the method of contour integration prove that  $\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)^2} dx$
- ii. Determine the poles and the residues at each pole of the function  $f(z) = \cot Z$  **(Nov 10)**
47. i. Find the Residues of  $f(z) = \frac{z^2}{z(z+2)^3}$  at  $Z = -2$
- ii. Find the Residues of  $f(z) = \frac{z^3}{(z-1)^4(z-2)(z-3)}$  at  $|z|=1$  **(Nov 10)**

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*It is not a pleasant condition, but certainty is absurd.*

- Voltaire

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48. i. Define conformal Transformation. Show that a bilinear transformation is conformal.  
 ii. Show that circles are invariant under linear transformation  $w = az + c$ . **(Nov 10)**
49. i. State and prove Cauchy's Residue theorem. **(Nov 09,08)**  
 ii. Calculate the residue at  $z = 0$  of  $f(z) = \frac{1 + e^z}{z \cos z + \sin z}$ ,
50. i. Determine the poles of the function  $f(z) = z^2 / (z+1)2(z+2)$  and the residues at each pole. **(Nov 09,08)**  
 ii. Evaluate  $\oint_c \frac{dx}{(z^2+4)^2}$  where  $c = |z - i| = 2$
51. Find the poles of the function  $f(z) = \frac{1}{(z+1)(z+3)}$  and residues at these poles.
52. Show by the method of residues,  $\int_0^\pi \frac{d\theta}{a+b \cos \theta} = \frac{\pi}{\sqrt{a^2-b^2}}$  ( $a > b > 0$ ). **(Nov 09, 08)**
53. Evaluate  $\int_0^{2\pi} \frac{\sin^2 \theta d\theta}{a + b \cos \theta}$  **(Nov 09, Feb 07, May 2000)**
54. i. Find the residues of  $f(z) = \frac{z^3}{(z-1)^4(z-2)(z-3)}$  at  $z = 1$   
 ii. Show that  $\int_0^{2\pi} \frac{d\theta}{2 + \cos \theta} = \frac{2\pi}{\sqrt{3}}$  **(Nov 10)**
55. Evaluate  $\int_0^{2\pi} \frac{d\theta}{(5-3 \sin \theta)^2}$  using residue theorem. **(Nov 08)**
56. Show by the method of contour integration that  $\int_0^\infty \frac{\cos mx}{(a^2 + x^2)} dx = \frac{\pi}{4a^3} (1 + ma)e^{-ma}$  ( $a > 0, b > 0$ ). **(Feb 08)**
57. Evaluate by contour integration  $\int_0^\infty \frac{dx}{(1+x^2)}$ . **(Feb 08)**
58. i. Evaluate by residue theorem  $\int_0^{2\pi} \frac{d\theta}{(2 + \cos \theta)}$   
 ii. Use the method of contour integration to evaluate  $\int_{-\infty}^\infty \frac{x^2 dx}{(x^2 + a^2)^3}$ . **(Feb 08)**
59. i. Show that  $\int_0^{2\pi} \frac{d\theta}{a + b \sin \theta} = \int_0^{2\pi} \frac{d\theta}{a + b \cos \theta} = \frac{2\pi}{\sqrt{a^2 - b^2}}$ ,  $a > b > 0$  using residue theorem.

*Doubt is not a pleasant condition, but certainty is an absurd one.*

- Voltaire

- ii. Evaluate  $\int_0^{\infty} \frac{dx}{(x^2 + a^2)^2}$  **(Feb 08, 07, Nov, May 04)**
60. Find the poles and residues at each pole of  $f(z) = \frac{\sin^2 z}{(z - \frac{\pi}{6})^2}$  **(Feb 08, Nov 06)**
61. Find the poles and residues at each pole of  $\frac{ze^z}{(z-1)^3}$  **(Feb 08, Nov, May 06)**
62. Determine the poles of the function  $\frac{z+1}{z^2(z-2)}$  and the corresponding residues **(Feb 08, May 06)**
63. Evaluate  $\oint_c \frac{dz}{\text{Sinh}z}$ , where  $c$  is the circle  $|z| = 4$  using residue theorem. **(Feb 08, May 05)**
64. Show that  $\int_0^{\pi} \frac{\text{Cos}2\theta}{1 - 2a\text{Cos}\theta + a^2} = \frac{\pi a^2}{\sqrt{1-a^2}}$ , ( $a^2 < 1$ ) using residue theorem. **(Feb 08, May 05)**
65. Evaluate  $\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + 1)(x^2 + 4)} dx$  **(Feb 08, Nov 07, May 04, 01 Jan 03)**
66. Determine the poles of the function  $\frac{(2z+1)^2}{(4z^3 + z)}$  and the corresponding residues **(Nov 07, May 05)**
67. Evaluate  $\int_c \frac{(\sin \pi z^2 + \cos z^2) dz}{(z-1)^2(z-2)}$  where  $c$  is the circle  $|z| = 3$  using residue theorem. **(Nov 07, May 05)**
68. Evaluate  $\int_0^{2\pi} \frac{d\theta}{(a + b \cos \theta)^2}$ ,  $a > b > 0$ , using residue theorem. **(Nov 07, May 05)**
69. Using the complex variable technique evaluate  $\int_{-\infty}^{\infty} \frac{dx}{x^4 + 1}$  **(Nov 07, May 2000)**
70. Evaluate  $\int_c \frac{ze^z dz}{(z^2 + 9)}$  where  $c$  is  $|z| = 5$  by residue theorem. **(Nov 07)**
71. Evaluate  $\int_0^{2\pi} \frac{\text{Cos}2\theta}{5 + 4\text{Cos}\theta} d\theta$  using residue theorem. **(Nov 07)**
72. Evaluate  $\int_0^{\infty} \frac{dx}{(x^2 + 1)^3}$  using residue theorem. **(Nov 07)**

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*One can survive everything, nowadays, except death, and live down everything except a good reputation.*

- Oscar Wilde

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73. Evaluate  $\int_C \frac{ze^z dz}{(z^2 + 9)}$  where C is  $|z| = 5$  by residue theorem. (Nov 07)
74. Evaluate  $\int_0^\alpha \frac{dx}{(x^2 + 1)^3}$  (Nov 07, May 05)
75. Show that  $\int_0^\pi \frac{a d\theta}{a^2 + \sin^2 \theta} = \frac{\pi}{\sqrt{1+a^2}}$ , ( $a > 0$ ) using residue theorem. (Nov 07, 04)
76. Determine the poles and the corresponding residues of  $f(z) = \frac{z+1}{z^2(z-2)}$  (Feb 07)
77. Evaluate  $\oint_C \frac{dz}{\sinh z}$ , C:  $|z| = 4$  using residue theorem (Feb 07)
78. Evaluate by Countour integration  $\int_0^\infty \frac{dx}{1+x^2}$  (Feb 07)
79. Find the poles and the residues at each pole of  $f(z) = \frac{1-e^z}{z^4}$  (Feb 07)
80. Evaluate  $\int_C \frac{z-3}{z^2+2z+5} dz$  where C is the circle (i)  $|z|=1$  (ii)  $|z+1-i|=2$  using residue theorem (Feb 07)
81. Evaluate by residue theorem  $\int_0^\infty \frac{dx}{x^6+1}$  using residue theorem (Feb 07)
82. Evaluate  $\int_0^{2\pi} \frac{d\theta}{(5-3\cos\theta)^2}$  using residue theorem (Feb 07)
83. Evaluate  $\int_0^\infty \frac{\sin mx}{x} dx$  by residue theorem (Feb 07)

## UNIT-V

- a) Show that the function  $w = \frac{z-i}{z}$  transforms the straight line  $x=c$  in the z-plane in to a circle in the w-plane.
1. b) Under the transformation  $w = \frac{z-i}{1-iz}$  find the image of the circle (Dec 13)
- (i)  $|w|=1$  (ii)  $|z|=1$  in the w-plane. [15]

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*One should always play fairly when one has the winning cards.*

- Oscar Wilde

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2. a) Show that the transformation  $W = z^2$  maps the circle  $|z-1|=1$  into the cardioid  $\rho = 2(1 + \cos \theta)$  where  $W = \rho e^{i\phi}$
- b) Find the bilinear mapping which maps the points  $z=-1, -i, -1-i$  to  $w=1, 0, -i$  (May13)
3. a) Find the image of the strip  $\frac{-\pi}{2} < x < \frac{\pi}{2}; 1 < y < 2$  under the mapping  $w(z) = \sin(z)$ . Draw the rough sketch of the regions in Z-plane and W-plane.
- b) Find the bilinear transformation that maps the points  $z_1 = -1, z_2 = 0, z_3 = 1$  in to the points  $w_1 = -1, w_2 = 1, w_3 = 1+i$  respectively. (Dec 12)
4. a) find the image of the circle  $|z-2i|=2$  in the complex plane under the mapping  $w = \frac{1}{z}$ .
- b) Show that the transformation  $w = \frac{i(1-z)}{1+z}$  transforms the circle  $|z|=1$  in to the real axis in the w-plane and the interior of the circle into upper half of the w-plane. (Dec 12)
5. Find the bilinear transformation which maps  $z_1 = 1 - 2i, z_2 = 2 + i, z_3 = 2 + 3i$  in to the points  $w_1 = 2 + 2i, w_2 = 1 + 3i, w_3 = 4$  respectively. Find the fixed and critical points. (Dec 11)
6. i. Show that the transformation  $w = \cos z$  maps the half of the z-plane to the right of the imaginary axis into the entire w-plane.
- ii. Show that the transformation maps the family of line parallel to y-axis to ellipse. (Dec 11)
7. i. Find the points at which  $w = \cosh z$  is not conformal.
- ii. Find the image of the strip bounded by  $x = 0$  and  $x = \pi/4$  under the transformation  $w = \cos z$ . (May 11)
8. Find the bilinear transformation which maps  $z_1 = 1; z_2 = i; z_3 = -1$  in to the points  $w_1 = i; w_2 = 0; w_3 = -i$  respectively. Find the fixed and critical points of this transformation and find the image of  $|z| < 1$ . (May 11)
9. i. Find the image of the triangle with vertices at  $i, 1+i, 1-i$  in the z-plane, under the transformation  $e^{\frac{5\pi i}{3}}(z-2+4i)$ .
- ii. Find the image of the infinite strip,  $0 < y < 1/2$  under the mapping function  $w = 1/z$ . (May 11)
10. i. Show that the transformation  $w = \frac{3-z}{z-2}$  transforms the circle  $|z - \frac{5}{2}| = \frac{1}{2}$  in the z-plane in to the imaginary axis in the w-plane.
- ii. For the mapping  $w = 1/z$ , find the image of the family of circles  $x^2 + y^2 = ax$ , where  $a$  is real. (May 11)
11. i. Find the map of the circle  $|z|=C$  under the transformation  $w = z - 2 + 4i$
- ii. Show that both the transformations  $w = \frac{z-i}{z+i}$  and  $w = \frac{i-z}{i+z}$  transforms  $|w| \leq 1$  into upper half plane  $\text{Im}(z) > 0$ . (Nov 10)
12. i. Find the image of  $|z|=2$  under the transformation  $w = 3z$ .
- ii. Show that the transformation  $w = z^2$  maps the circle  $|z-1|=1$  cardioid  $r = 2(1 + \cos \theta)$  (Nov 10)

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*Search others for their virtues, thyself for thy vices.*

- Benjamin Franklin

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13. i. State that the transformation  $W=e^z$  transform the region between the real axis and the line parallel to the real axis at  $y=\pi$  into upper half of the  $W$ -plane.  
 ii. Find the bilinear transformation which maps  $Z=-1, i, 1$  into the point  $W=-i, 0, i$ . **(Nov 09)**
14. Define conformal mapping? State and prove the sufficient condition for  $W = f(z)$  to be conformal at the point  $z_0$ . **(Nov 09 07, Jan 03)**
15. Find and plot the image of triangular region with vertices at  $(0,0), (1,0), (0,1)$  under the transformation  $w=(1-i)z+3$ . **(Nov 09,06, May 05, Nov 04)**
16. i. Find the image of the region in the  $z$ -plane between the lines  $y = 0$  and  $y = \frac{\pi}{2}$  under the transformation  $w = e^z$ . **(Nov 09, May 06)**  
 ii. Find the bilinear transformation which maps the points  $(-1, 0, 1)$  into the points.  $(0, i, 3i)$
17. Under the transformation  $w = \frac{1}{z}$  find the image of the circle  $|z-2i|=2$ . **(Nov 09, May 06)**
18. Show that the transformation  $w = z+1/z$ , converts that the radial lines  $\theta = \text{constant}$  in the  $z$ -plane in to a family of confocal hyperbolar in the  $w$ -plane. **(Nov 08)**
19. i. Find and plot the image of the regions  
 a.  $x > 1$   
 b.  $y > 0$   
 c.  $0 < y < 1/2$  under the transformation  $w = 1/z$   
 ii. Prove that every bilinear transformation maps the totality of circle and straight lines in the  $z$ -plane on to the totality of circles and straight lines in the  $w$ -plane. **(Nov 08)**
20. i. Show that horizontal lines in  $z$ -plane are mapped to ellipses in  $w$ -plane under the transformation  $w = \sin z$ .  
 ii. Define Bilinear transformation. Determine the Bilinear transformation which maps  $z = 0, -i, 2i$  into  $w = 5i, oo, -i/3$ . **(May 11, Nov 08)**
21. Find the image of the infinite strip  $0 < y < 1/2$  under the transformation  $w = \frac{1}{z}$ . **(Nov 08, 07, Feb 08, 07, May 05)**
22. Show that the image of the hyperbola  $x^2 - y^2 = 1$  under the transformation  $w = \frac{1}{z}$  is  $r^2 = \cos 2\theta$ . **(Nov 10)**
23. i. Show that the transformation  $w = i(1-z)/(i-z)$ , maps the interior of the circle  $|z|=1$  in to the upper half of the  $w$ -plane, the upper semi circle into positive half of real axis and lower semi circle into negative half of the real axis.  
 ii. By the transformation  $w = z^2$  show that the circle  $|z-a| = c$  ( $a$  and  $c$  are real) in the  $z$  plane correspond to the limacons in the  $w$ -plane. **(Feb 08)**
24. Find the bilinear transformation which maps the points  $(-1, 0, 1)$  into the points  $(0, i, 3i)$ . **(Feb 08, 07, Nov 07, May 05)**

A fanatic is one who can't change his mind and won't change the subject.

- Sir Winston Churchill

25. Find the image of the domain in the  $z$ -plane to the left of the line  $x = -3$  under the transformation  $w = z^2$   
(Feb 08, Nov 06)
26. Find the bilinear transformation which maps the points  $z = 2, 1, 0$  into  $w = 1, 0, 1$  respectively.  
(Feb 08, Nov 06)
27. Show that the transformation  $w = \frac{2z+3}{z-4}$  changes the circle  $x^2 + y^2 - 4x = 0$  into the line  $4u+3=0$   
(Feb, Nov 07)
28. Find the bilinear transformation which maps the points  $(0, 1, \infty)$  into the points  $(-1, -2, -i)$ .  
(Nov 07, 06, May 06, 2000)
29. Let  $f(z)$  be analytic function of  $z$  in a domain  $D$  of the  $z$ -plane and let  $f'(z) \neq 0$  in  $D$ . Then show that  $w = f(z)$  is a conformal mapping at all points of  $D$ .  
(Nov 07, 03)
30. Find the bilinear transformation which maps the points  $(-i, 0, i)$  into the point  $(-1, i, 1)$  respectively. (Nov 07)
30. Under the transformation  $w = \frac{z-i}{1-iz}$ , find the image of the circle  $|z| = 1$  in  $w$ -plane. (Feb 07)
31. Show that the mapping  $w = z + \frac{1}{z}$  maps the circle  $|z| = c$  into the ellipse  $u = (c + \frac{1}{c}) \cos \theta$ ,  
 $v = (c - \frac{1}{c}) \sin \theta$ . Also discuss the case when  $c = 1$  in detail. (Feb 07)

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*Although prepared for martyrdom, I preferred that it be postponed.*

- Sir Winston Churchill

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*Broadly speaking, the short words are the best, and the old words best of all. To profit from good advice requires more wisdom than to give it.*

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- John Churton Collins